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Pre-service teachers manifested mathematics pedagogical content knowledge: The role of the teaching practicum

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ABSTRACT

Pedagogical content knowledge (PCK) begins to develop during teacher education and the teaching practicum reinforces its development. This study is an interpretive inquiry into pre-service mathematics teachers' practicum classrooms. The study explored four pre-service mathematics teachers' PCK through observations and interviews. The pre-service teachers were found teaching various mathematics topics to students at different levels of secondary education. Findings indicate that pre-service teachers demonstrated some important elements of mathematics PCK. However, they tended to focus more on procedural rather than conceptual learning, lacked knowledge of some topic-specific teaching and learning resources, and made some flaws, possibly emanating from a lack of classroom experience. The study recommends teacher education to focus more on analysis of the mathematics curriculum and opportunities for practical teaching sessions including practicum and microteaching that are supported by experienced mentors in mathematics.

Keywords: mathematics, PCK, pre-service teachers, teacher education, teaching practicum

INTRODUCTION

The teaching of mathematics requires teachers to possess knowledge of both mathematics and the ways of facilitating the learning of mathematics. The former may be referred to as knowledge of the subject matter or content knowledge (CK) and the latter may be referred to as the pedagogical content knowledge (PCK) (Kadarisma et al., 2019). The two do not operate in isolation but in a complex interaction that has to be developed over time through teacher education and practice or experience (Anney & Bulayi, 2020; Copur-Gencturk & Li, 2023; Moh'd et al., 2021). The mathematics PCK (M-PCK) may be considered less theoretical and more practical (Jeschke et al., 2019; Maniraho & Christiansen, 2015b). It is a knowledge developed and demonstrated through practice over a period of time (Copur-Gencturk & Li, 2023; Kadarisma et al., 2019). Various studies have pointed out the role of experience in M-PCK where evidence shows that the more experience the higher the M-PCK (Anney & Bulayi, 2020; Copur-Gencturk & Li, 2023; Moh'd et al., 2021) although this is sometimes not significant (Maniraho & Christiansen, 2015b).

With an understanding of the role of practical experience in developing PCK (Kyndt et al., 2016) teacher education programs have a teaching practicum component (Kadarisma et al., 2019). The teaching practicum places pre-service teachers in the actual classroom as teachers in the making under the guidance and support of experienced teachers and university or college supervisors (Kadarisma et al., 2019). It allows pre-service teachers to learn teaching through doing. The developed experience is not only expected to widen and deepen their understanding of the subject matter (Copur-Gencturk & Li, 2023), but also a way through which learners may effectively learn the subject matter (Anney & Bulayi, 2020). It is through this experience that pre-service teachers develop the knowledge of teaching specific subject matter.

Although learning through practice is recognized as important by many (Copur-Gencturk & Li, 2023; Jeschke et al., 2019), the nature of this knowledge as demonstrated in the classroom has received limited attention especially the subject-specific knowledge for effective classroom delivery (Copur-Gencturk & Li, 2023; Jeschke et al., 2019; Maniraho & Christiansen, 2015b). The limited attention to the understanding of the knowledge and skills pre-service teachers develop or possess as a result of undertaking teaching practicum has various effects. First, it limits the understanding of the effectiveness of the teaching practicum in developing pre-service teachers' competences to teach (Copur-Gencturk & Li, 2023). Second, it hampers the informed decisions and planning the ways of helping or improving the teaching practicum experiences (Moh'd et al., 2021) for the pre-service teachers. Third, it limits the ability to plan and provide focused mentoring activities for the development of specific competences that pre-service teachers may need (Kyndt et al., 2016). Fourth, it contributes to limited awareness of the nature and level of the competences pre-service teachers possess and the support needed.

Pedagogical Content Knowledge

Mathematics teaching requires the knowledge of the subject–mathematics CK (M-CK) and the knowledge of teaching mathematics–M-PCK. Both M-CK and M-PCK are developed through university or college mathematics courses and specific courses for mathematics instruction (Depaepe et al., 2013; Jeschke et al., 2019; Kadarisma et al., 2019). As pointed out earlier, the blend between the two is more practical than theoretical and hence must be developed through practice. It is a domain-specific instructional knowledge (Copur-Gencturk & Li, 2023) that Jeschke et al. (2019) analyzed as comprising three requirements: subject matter; effective means of delivering or developing the subject matter; and the application of the two in the actual classroom situation within time and contextual constraints. Inquiry into this knowledge can be stressed back to Shulman (1986) in the study "Those who understand: Knowledge growth in teaching". Shulman (1986) argued that teachers need to possess PCK which is the practical interaction of CK and pedagogical knowledge. Shulman (1986) categorized teacher knowledge into three categories: CK, PCK, and curricular knowledge. Shulman (1986) defined CK in line with Schwab (1978) as being substantive (having concepts and principles) and the syntactic (procedural) structures. Shulman (1986) also defined PCK with reference to the subject matter indicating the centrality of the CK as "the most regularly taught topics in one's subject area, the most useful forms of representations of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations" (p. 9). Curricular knowledge was conceptualized as a set of programs or activities for teaching a topic, a set of relevant instructional materials or resources for the topic, and a set of indicators or characteristics for the use of activities and resources (Shulman, 1986). This concept has been widely adopted, reconceptualized, and contextualized in different studies. This has led to the expansion of the span of PCK components over the years (Moh'd et al., 2021).

Although the increased number of PCK components has continually reformed the inquiry into the teacher knowledge, some scholars (Carlson et al., 2019; Hume et al., 2019) have thought to re-examine it and establish consensus in ways the teacher knowledge may be viewed. This led to the suggestion that PCK may be viewed as three dependent components namely enacted PCK (ePCK), personal PCK (pPCK), and collective PCK (cPCK) (Carlson et al., 2019). The cPCK informs the pPCK which then informs the ePCK (Botha et al., 2023). The current study adopts the ePCK where pre-service mathematics teachers were observed when teaching and interviewed for their views about their practice.

Mathematics Pedagogical Content Knowledge

Knowledge of mathematics subject matters (M-CK) is indispensably important for mathematics teachers (Copur-Gencturk & Li, 2023). This knowledge involves the substantive and the syntactic structures (Schwab, 1978). The substantive structures in mathematics include mathematical concepts and principles and their organizations, and the syntactic structures refer to the established mathematical procedures and rules for arriving at or justifying substantive mathematical structures. M-CK is necessary for the development of mathematics PCK (Jeschke et al., 2019). However, when this knowledge is considered apart from learners, the learning process, and resources become useless pedagogically (Ball et al., 2008; Shulman, 1986).

To address this, Ball et al. (2008) analyzed Shulman's (1986) model with more attention to the knowledge that mathematics teachers need for teaching. In their study they synthesized two more categories from the CK into CCK–common CK (Knowing how to calculate or arrive at a correct mathematical answer and sources of error in the mathematical algorithms that might have been committed by learners, independent of knowing the learners) and SCK–specialized CK (knowing the specific elements of mathematical concepts, talking using mathematical language, explaining, and justifying mathematical ideas) that are specifically needed for teaching (Li, 2020). They also split the PCK into two that is KCS–knowledge of content and students (knowing the mathematics content and how students tend to interact with it) based on experience of previous lessons (Botha et al., 2023) and KCT–knowledge of content and teaching (knowing how to organize learning activities and mathematical representations for the development of the intended mathematics concepts). Ball et al. (2008) argued that M-CK for teaching does not necessarily require knowledge of students or teaching. However, in practice, to make use of it, teachers must have enough knowledge of students and teaching (Ball et al., 2008).

Some studies on M-PCK have looked at different categories including knowledge of students' understanding and the knowledge of instructional strategies (Aliustaoğlu et al., 2019; Fukaya & Uesaka, 2023). Li (2020) focused on SCK, KCS, and KCT leaving out the CCK and curricular knowledge. Aliustaoğlu and Tuna (2019) define knowledge of students understanding as the preliminary comprehension of the students about the subject matter, students' mistakes and misconceptions, and their possible reasons or sources. This is similar to KCS as explained by Ball et al. (2008). They also referred to knowledge of instructional strategies as knowing the demonstrations, examples, and explanations for resolving mistakes and misconceptions. This may also fit well with KCT by Ball et al. (2008). Nevertheless, instructional strategies refer to ways of facilitating the construction of new knowledge in addition to correcting mistakes or misconceptions. Knowledge of the subject matter is less debated in PCK as may be seen in studies by Aliustaoğlu and Tuna (2019) and Fukaya and Uesaka (2023), however, it is considered important in this study since preserve teachers were not only expected to demonstrate it but also because the study aimed to assess whether they demonstrated its mastery. Copur-Gencturk and Li (2023) point out that knowledge of subject matter plays a key role in the understanding of students' mathematical thinking.

Research Questions

1. The study explores pre-service mathematics teachers' PCK as demonstrated during their teaching practicum. It specifically attends to the following research questions:
2. How do pre-service teachers demonstrate knowledge of the subject matter?
3. How do pre-service teachers demonstrate knowledge of pedagogical resources relevant to their topic?
4. What curriculum resources do pre-service teachers use in facilitating mathematics learning?

5. What deliberate efforts do pre-service teachers make to address potential learning difficulties in the classroom?
6. These questions provide a basis for further inquiry about the extent to which pre-service teachers develop knowledge and skill through teaching practicum and hence implications for teacher education (Copur-Gencturk & Li, 2023).

METHODOLOGY

Study Design

This study is an interpretive inquiry into four classroom cases where four pre-service mathematics teachers were observed when teaching and interviewed thereafter about their classroom sessions. Observations were done to collect data about classroom activities as performed by the pre-service teachers. This was intended to obtain evidence of their demonstrated M-PCK as they interacted with students and teaching and learning resources. The observations were followed by reflective interviews to elicit pre-service teachers' opinions, understanding, justification, and reasons for their actions that were observed during the classroom sessions.

Context of the Study

Participants of the study were pre-service mathematics teachers. They were pursuing a Bachelor of Science with education. During the time of data collection, they were undertaking their teaching practicum in four different schools. The practicum was done for eight weeks where pre-service teachers were expected to practice teaching in real classrooms. The context of the practicum sites is reported in the literature and experience of the researcher as having limited mentoring from experienced teachers (Mahende & Mabula, 2014; Njiku, 2018). Similarly, support from university supervisors is always limited as they do not have sufficient time to stay and interact with pre-service teachers in the placement schools. Pre-service teachers have to forge their way ahead into the unknown of the teaching practicum, a situation which may hinder their development not only professionally but also in PCK.

The teaching of mathematics requires not only the knowledge of the required resources but also their access (Sujadi et al., 2019). Because pre-service teachers are relatively new to the schools, they are faced with a high need to familiarize themselves with the environment. Furthermore, as teachers in the making, they have a lot to learn from experienced teachers and their own experience. Simultaneously, they still have to deliver lessons sometimes with limited mentoring from both school-based mentors and university supervisors. Thus, this study is situated in the classroom of pre-service mathematics teachers to examine their teaching and indicators of M-PCK.

The four pre-service teachers were all teaching mathematics in different class levels of secondary education. PT1 taught algebra to form-one students. The subtopic was simultaneous equations, and the lesson was on elimination methods of solving two unknown simultaneous equations. PT2 taught logarithms to form-two students and the subtopic was tables of logarithms. The lesson was on antilogarithms of numbers. PT3 taught the topic of the earth as a sphere to form-three students and the subtopic was distances along great circles. The lesson was focused on calculating distances along great circles. PT4 taught rates and variation to form-two students. The subtopic was direct variation, and the lesson was on graphs of direct variation.

Data Collection

The study was qualitative, hence qualitative data was collected through classroom observations and interviews. Observations were made once in the classrooms where pre-service teachers taught. Each lesson was forty minutes long. They were done focusing on how pre-service teachers connected students' prior knowledge with the current lesson, lesson delivery and the kind of resources used, learner engagement and interaction, and learning activities given and managed. Data was collocated through note-taking where important details were recorded in the notebook in the form of a story. Interviews were done after classroom sessions. The interviews were conducted to collect students' understanding of the topic and students. The items were focused on content or activities that would potentially be challenging to the students and possible ways of addressing them, and knowledge of the mathematics curriculum they implemented. Interview data was taped using an audio recorder.

Data Analysis

Observation data that were taken using a notebook were later typed into the computer for analysis. Similarly, the audio-recorded interview data were later transcribed into text on the computer. This process of committing data into text served the purpose of data familiarization since it involved reading the notebook and listening to the audio records several times while typing. The transcripts were then imported into the MaxQDA for coding. The initial analysis generated fifteen codes which were finally discussed into six themes. The themes were presented together with teaching aid images and some mathematical expressions or equations as were observed in the classroom. The analysis was made in such ways that PCK episodes, particularly the M-CK and M-PCK are identified and explained in context. The episodes were identified first from observation and supported by interview data.

FINDINGS AND DISCUSSION

Pre-Service Mathematics Teachers Guide Students to Make Use of Prior Knowledge

During classroom observation, pre-service teachers were found to begin their lessons by asking their students about what they had learnt in their previous lessons. All four pre-service teachers used questions similar to "What did we learn in the previous lesson?" The importance of requiring students to recall what they had learnt in previous lessons would be seen in the activities that followed. For example, PT2 needed to use character and mantissa as key terminologies in the lesson on antilogarithms of numbers, and students mentioned these terms as concepts learnt in the previous lessons. Similarly, in the lesson about variation, students responded that they had learnt about direct variation. The pre-service teacher explained that in the current lesson, they will build on direct variation and wrote on the board the variation $y \propto x$ which leads to $y = kx$ to the drawing graphs of direct variation. Additionally, PT3 used the concept of angles subtended at the center of the earth by lines from points on the surface as responded by students to add that in the current lesson they will use such angles to calculate the distance between two points on the surface of the earth. This suggests that pre-service teachers believed and made use of previously learnt knowledge in developing new knowledge.

These findings indicated that when introducing the lesson, pre-service teachers were found to guide students to reflect on the previous lessons. Although they did not use a variety of questions that would be concept-based including asking students their understanding of specific concepts at the beginning of the lesson, they seem to value the role of connecting between lessons (Maniraho & Christiansen, 2015a). The use of prior knowledge by teachers when teaching mathematics has also been explored by Botha et al. (2023), Makonye (2020), Maniraho and Christiansen (2015a), and Wicaksono and Dwipa (2020). These studies evidence that although there could be diverse ways of using what students already know, teachers tend to consider such knowledge important in their lessons (Maniraho & Christiansen, 2015a). It is argued that knowing the level of proficiency in a specific mathematics topic allows teachers to pre-plan and pitch the lesson to the appropriate level of students understanding (Botha et al., 2023). However, teachers must use such knowledge to inform the next topic rather than simply for the matter of recalling back (Maniraho & Christiansen, 2015a). Similarly, knowing the breadth and depth of the content to be taught to students is important (Wicaksono & Dwipa, 2020) since it allows teachers to negotiate between students' abilities and the learning activities for particular concepts. It also allows the integration of existing beliefs and cultural knowledge which may be used to build upon or must be negotiated with (Makonye, 2020). To effectively deliver lessons that benefit from students' prior knowledge, teachers must think about how students are likely to engage with the subject matter and the needed link between what they know and what they don't know (Fukaya & Uesaka, 2023; Makonye, 2020).

Pre-Service Mathematical Teachers Believe in Step-by-Step Processes

One of the characteristics of mathematics is its procedural nature. It is also important to note that mathematics is necessarily conceptual. It is not merely about finding the right answer but the process to the right answer is what depicts mathematical thinking. This was also observed in pre-service students' mathematics lessons. Observations show that pre-service teachers guided students stepwise to arrive at solutions to mathematical problems embedded in concepts they were to learn. For example:

- PT2 explains that the inverse of logarithms is also called antilogarithms or in short antilog.
- That is $log^{-1} = antilog$.
- He then writes while explaining that suppose a number $x = 1.8810$.
- To find the antilog of a number one should divide it by log hence $x/log = 1.8810/log$ and this leads to the antilog of $x = log^{-1} 1.8810$.
- He then asks students what they need in order to arrive at the antilog of a number, they raise hands, and one is selected. The student mentions that they need to determine the character and mantissa.
- He then explains that from the number in question, the character becomes the power of ten and the mantissa has to be read from tables.
- He then goes ahead to guide students on how to read the mantissa from the tables.

Similar procedures are demonstrated by other pre-service teachers. In some cases, they end their lessons by asking students to summarize the stages they went through. During the interview, PT1 submitted that this was important to help students master the stages that would help them in solving similar mathematical problems. However, critical discussions or questions for the development of conceptual understanding were limited.

Mathematics is both a procedural and conceptual subject and pre-service teachers were found to guide students step by step into concept development. A logical flow of ideas or arguments is important for students to follow. When the flow of arguments is important, teachers tend to focus on the procedure (Botha et al., 2023) and tend to follow a sequence of stages proposed in textbooks (Bologna et al. 2022). Botha et al. (2023) report that more explanations were used by teachers to develop conceptual understanding in addition to mere procedural knowledge. Stepwise explanation or detailed procedure is reported in the study by Weber et al. (2023) to reduce abstraction while maintaining mathematical integrity. However, similar to this study Bologna et al. (2022) and Botha et al. (2023) found that thoughtful oral discourse was lacking, hindering the development of conceptual understanding. This study suggests that teachers may need to pay attention to what the arguments mean and the way arguments are presented in mathematics which in turn does not only allow students to follow but to master the reasoning inherent in the concepts being taught. Conceptual understanding is more important than mere memorization of calculative procedures in the development of further competences (Bologna et al., 2022; Botha et al., 2023). For this, unlike the findings of the current study,

Kadarisma et al. (2019) found teachers used inquiry learning as used in the scientific process to teach the concepts of quadrilaterals. The scientific process allows the development and harmonization of diverse mathematical experiences. Such experience may however be informed by learners' beliefs and culture which Makonye (2020) argues must be negotiated for effective conceptual understanding where teachers should possess sufficient CK to inform their use of multiple representations and good examples.

Pre-Service Mathematics Teachers Recognize That Practice is Imperative in Mathematics

The teaching of mathematics was mainly done through working out specific conceptual examples where terminologies and their meanings were part of the whole process or a prerequisite and hence were used as communicative resources. PT1 for example, used two simultaneous equations to guide learners through the elimination method while PT2 added more examples where he guided students to find numbers whose logarithms are for example a: 3.7515 and b: 1.2466 and step by step guided students to arrive at $x = 5643$ for 3.7515. PT4 begins with $y \propto x$ to guide students into obtaining values of x- and y-intercept and then adds yet another variation $y \propto x + a$. In addition to examples where the teacher orchestrates the process, exercises were used mainly when students sat in groups. Pre-service teachers used group work activities to assign students exercises where they would work collaboratively. For example, PT4 asked students to draw a graph of $x = y$ while PT1 gave a pair of simultaneous equations to students for them to find the values of the unknowns. All four pre-service teachers provided homework for further practice before concluding their lessons.

Interview data shows that pre-service teachers saw more time, including remedial sessions, to be important for mastering mathematical concepts. They also believed that more exercises would enable students to develop conceptual understanding of mathematics.

Practice makes perfect and mathematics requires lots of it (Ma'rufi & Ilyas, 2019). Pre-service teachers were found to reinforce the learned concepts by providing examples and exercises. Giving time to students to try and practice solving mathematical problems is reported in various studies where group work activities provided students with the opportunity to engage with learning scenarios or content instead of simply listening to the teacher (Kılıç, 2011; Ma'rufi & Ilyas, 2019; Metscher et al., 2021). On the contrary, Maniraho and Christiansen (2015a) found that when mathematics teachers use whole-class interaction or individualized work, they become less effective. Similar to this study where pre-service teachers used group work for collaborative learning, Correia and Baptista (2022) and Metscher et al. (2021)found that pre-service teachers were found to use learner centered approaches including cooperative learning, where learners work on exercises in groups among other learner centered approaches common to STEM subjects. This study found that pre-service teachers valued the role of practicum, and some would spend even more time (Kılıç, 2011; Ma'rufi & Ilyas, 2019) such as remedial sessions for students lagging.

Pre-Service Mathematics Teachers' Understanding of Curriculum Materials

Common curriculum materials prepared by pre-service teachers include lesson plans, scheme of works, and lesson notes which also include examples and exercises. Other materials included charts as used by PT4 to show a graph of a variation. However, the chalk and chalkboard were mostly used including drawing diagrams as used by PT3 when showing students how to calculate the distance between two points on the surface of the earth. However, pre-service teachers could not list a wide span or range of materials relevant to the topic they taught in comparison to what was listed in the syllabus. For example, PT4 listed manila paper for sketching graphs, lesson plans, and notes while the syllabus had more to include such as graph papers/board, mathematical instruments, colored chalks, geoboard, and rubber bands (Tanzania Institute of Education, 2005). PT3 listed the globe and ruler while the syllabus listed the globe, oranges, watermelon, models of a skeleton sphere and soft grass, atlas, colored chalks, graphs papers, magnetic compass, compasses, ruler, thread, and protractor. PT1 mentioned manila paper and marker pens, notes, and past examination papers while the syllabus had more such as manila paper, marker pens, colored chalk, and worksheets. PT2 listed mathematical tables and books, yet manila papers, colored chalks, marker pens, charts of logarithmic laws, and logarithmic tables were in the syllabus.

The use of teaching materials such as drawings or charts must have a rationale. Pre-service teachers showed diverse understanding especially when providing justifications for their use. PT4 used a chart with a graph drawn on it. He explained that he wanted students to see what the graph of $y = x + a$ looks like (see **Figure 1**). Not only that this was contrary to the lesson objectives of enabling students to be able to draw, but it was also observed that the display of the graph was done after students had done several examples of drawing in groups. PT3 on the other hand drew a figure (see **Figure 2**) on the board and used it to guide students to calculate the distance of two points on the earth's surface using angles they subtend at its center. During the interview, he explained that the figure worked better than a globe which would not have allowed students to see the angles at its center.

Curriculum conception is at the heart of teacher education and pre-service teachers are expected to demonstrate mastery (Sujadi et al., 2019). In addition to the CK, mathematics curriculum knowledge includes resources or materials specific to the topics (Shulman, 1986; Sujadi et al., 2019). This study found that pre-service teachers had some understanding of the curriculum resources for the topics they taught. For example, the use of a three-dimensional drawing instead of a globe is evidence of the understanding of multiple representations of the concept to develop conceptual understanding. Appropriate and detailed representation of concepts is explained by the way teachers model or illustrate concepts including the use of materials and diagrams (Makonye, 2020; Sujadi et al., 2019;). The use of learning media may not only be used for multiple representations but also for addressing language problems, especially in contexts similar to this study where the language of instruction is a foreign language (Sujadi et al., 2019) and beliefs and cultural conflict may be addressed by the use of realistic and context relevant resources (Makonye, 2020).

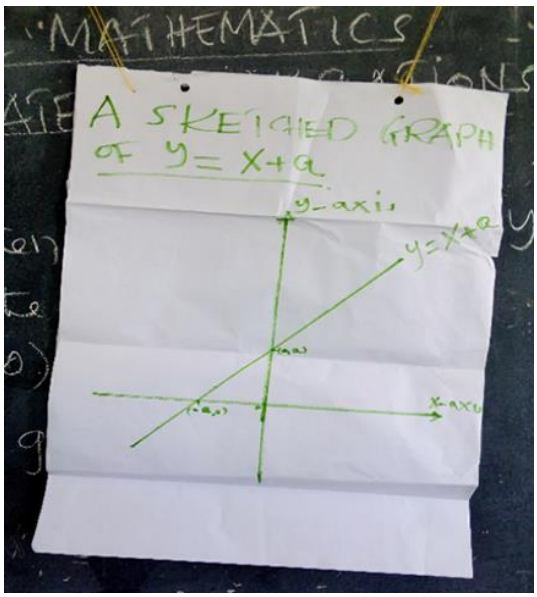

Figure 1. Graph of $y = x + a$ (Source: Field study)

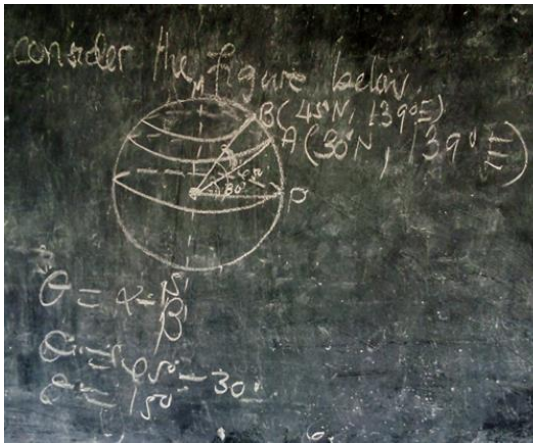

Figure 2. 3D image for explaining the globe (Source: Field study)

However, this study found that pre-service teachers needed more knowledge of subject-specific resources that align with the syllabus for them to effectively facilitate learning. It was observed that articulation of concepts and mastery of a range of pedagogical resources were still a problem for pre-service teachers. This may be developed through problem-based learning as part of the mathematic teaching methods course where pre-service teachers must explore and analyze resources needed for teaching specific concepts (Martin & Jamieson-Proctor, 2022). Knowledge and use of subject-specific curriculum materials are important as they not only contribute to the effectiveness of teaching but also to the development of teachers' PCK especially knowledge of mathematical thinking (Copur-Gencturk & Li, 2023; Martin & Jamieson-Proctor, 2022).

Pre-Service Mathematics Teachers' Knowledge of the Students' Difficulties

Pre-service teachers were asked to identify areas of the lesson students would find difficult. This is important because teachers should anticipate challenges that students may face and plan ahead how to address them. Although three of the four struggled to explain what they thought would be difficult to their students, it indicated that pre-service teachers did not master this crucial element in PCK. Nevertheless, after giving it a thought, PT4 comments that the drawing of the graph might have been the difficult part of the lesson possibly because it was the first time for the students to draw graphs. Similarly, PT3 submitted that since he drew a diagram on the board, students would find it difficult to associate the point on the surface of the earth, as shown on the figure as would be done if he had used the globe. Considering the two–diagram and the globe–both seem to have some added value, this might suggest he needed both of them, which he reluctantly admitted. However, PT1 explained that in solving simultaneous equations through the elimination method, making the coefficients of one of the unknowns the same was a difficult stage. He further explained that

> "when one is done with making the coefficients of one unknown the same, the rest of the stages are easy. This is because subtracting one equation from the other and making the remaining unknown the subject are things that students already know".

Pre-service teachers had two approaches to addressing difficulties and challenges that students might have faced during the lesson. One was through planned activities in the lesson and the other was through the after-class activities. PT4 submitted that he gave more time for students in groups to discuss and practice graphing, which he identified as a difficult part of the lesson. He further explains that he ensured this by passing through the groups to check, provide any needed help, and mark the completed tasks. PT3 thought that the globe would perfectly show points on the surface although it was not available in the school. Additionally, pre-service teachers were of the view that not all students would find such areas challenging. As such, they were planning, which seemed to be a common practice, to use extra time in the form of remedial sessions to help those who needed extra help.

Knowledge of students' difficulties in specific mathematics topics is important. This is supported by Aliustaoğlu and Tuna (2019 and Bologna et al. (2022) that determining learning areas where students might find challenging is a central component of PCK. When such difficulties are anticipated by teachers, they may enable them to plan ways of addressing them. Some pre-service teachers were able to identify areas students would have found difficult although they did not report having thought of them before the lesson. Identification of learning difficulties is important in teaching mathematics (Kılıç, 2011). For example, Weber et al. (2023) report pre-service teachers developing and appreciating their increased understanding of what their pupils can understand which might be attributed to their class level or age. Knowledge of students' mathematical thinking and hence knowledge of possible difficulties develops through teaching practicum (Copur-Gencturk & Li, 2023) suggesting the importance of sufficient time for teaching practicum. Although some students could explain areas where their students would find some difficulties, it was observed that the identified areas were not well thought out (Kılıç, 2011). This is contrary to what Fukaya and Uesaka (2023) suggest about the spontaneity of PCK application where teachers should spontaneously think about and predict possible misconceptions students might have and hence pre-plan to address them. Probably, this area of PCK was not well developed among pre-service mathematics teachers (Kılıç, 2011) who participated in the study. As a result, they could not articulate their pre-planned strategies to help their students address these difficulties, a finding similar to Aliustaoğlu and Tuna (2019) and Kılıç (2011) who found that despite being able to determine students' mistakes, pre-service teachers were unable to express properly the source and process of correcting such mistakes. The analysis found that pre-service teachers had a limited range of techniques to address students' learning difficulties. These were limited to the use of more time including remedial lessons and groupwork activities. Correia and Baptista (2022) attribute the limited use of some teaching and learning approaches such as engineering processes to little preparation of pre-service teachers.

Pre-Service Mathematics Teachers Can Make Flaws

During observation, some pre-service mathematics teachers could not explain some concepts accurately. Although this was not observed as being prevalent throughout the observed lessons, some explanations provided were inaccurate. For example: when teaching solving simultaneous equations, PT1 explained that after making the coefficient of one unknown the same, one should "*subtract equation one and two*" instead of subtracting equation one from two. No attempt was made to correct this explanation. The same pre-service teacher informs students that there are two ways to solve simultaneous equations namely substitutions and elimination methods, against the fact that the graphical method is also taught in the same grade level although in a different topic–coordinate geometry. PT3 when teaching the topic about the earth as a sphere explains that when two points subtend and angle at the center of the earth A^0 and B^0 then they have to be added, $A^0 + B^0$, when they are on different latitudes and longitudes, against the ideal explanation that addition should be done when the points are on different sides of the Equator or the Greenwich lines.

In some other cases preserves teachers provided examples and or used demonstration materials to develop mathematical concepts among their students. However, in some of these cases, such processes were not as would be expected. For example, PT4 used an example of a variation where $y \propto (x + a)$ which may be written in the form of an equation as $y = k\,(x + a)$, and when $k = 1$ then $y = x + a$. Then students are guided through this process up to when they obtain x and y intercepts where $(x, y) = (a, 0)$ and $(x, y) = (-a, 0)$. Students are then asked to plot on a graph paper the equation $y = x + a$. PT4 did not seem to recognize that in the equation $y = x + a$, "a" will remain unknown at the x and y intercepts and that this may make it difficult for students to come up with a graph. PT4 continued to show students a graph of $y = x + a$ as may be seen in **Figure 1**. Unfortunately, he did not take time to guide students on how to arrive at such a graph contrary to the specific competences intended to be developed during the lesson.

The use of visual illustrations like in **Figure 1** is also observed in the class by PT3. The earth draws an image akin to a sphere (**Figure 2**). However, **Figure 2** does not seem to be used in explaining the concept. PT3 and PT4 use these techniques as if they would speak on their own or with the thinking that students would figure out the process involved.

Teaching is a professional practice that needs fluency, and this develops over time. Novice teachers may find it challenging and this may result in pedagogical mistakes. Some mistakes or errors were so obvious suggesting that pre-service teachers need more practice possibly microteaching before embarking into real classroom practice. Teaching challenges are not uncommon among novice and pre-service teachers (Li, 2020), they happen even in experienced teachers' classrooms (Botha et al., 2023). For example, Botha et al. (2023) report a teacher using the symbol m for both meters and millimeters which might lead to confusion in students learning. The study found that pre-service teachers demonstrated limited understanding of the role of intended specific competence in determining what students were to learn and achieve from the lesson. Articulation of the learning purposes is an important PCK component and guides instruction to the achievement of the intended competences (Wicaksono & Dwipa, 2020). Wicaksono and Dwipa (2020) report pre-service teachers possess limited knowledge of the curriculum and have limited ability to state the purpose of their lessons appropriately in line with the curriculum. Li (2020) and Sujadi et al. (2019) added that teaching challenges have been observed even in the use of language specific to mathematics where teachers lack the vocabulary to present mathematical ideas.

Limitation

The study adopted a cross-sectional data collection limiting it from observing how practice changed over time. Longitudinal studies may provide more insight into the development of pre-service teachers' M-PCK as they negotiate through the tides of classroom dynamics. Also, the sample size was small limiting the study to interpretivism. Survey studies across schools involving more pre-service teachers might provide more data for population inferences.

Implication for Practice

To enable pre-service teachers to effectively teach mathematics, teacher education should provide pre-service mathematics teachers with mathematics pedagogical courses (Li, 2020). This would allow them to get familiar with mathematics-specific resources, language, and pedagogy. Mock teaching sessions such as microteaching may be used to develop pre-service teachers' attention towards students' mathematical thinking and potential difficulties students might face in various mathematics topics.

Pre-service teachers' practicum is a learning process and mentoring should be at the centre of it. Support, guidance, scaffolding, and feedback from experienced teachers are required. Schools, where mathematics pre-service teachers do their teaching practicum, should ensure that experienced mathematics teachers are assigned to provide mentorship. Reflective discussion between pre-service and experienced teachers may help to anticipate possible challenges and preplan potential solutions.

CONCLUSION

The study investigated pre-service mathematics teachers' PCK. This knowledge is important for effective teaching of the subject. The knowledge is developed during teacher education and advances during the career. Thus, teacher education plays a cardinal role in developing PCK. It introduces pre-service teachers to the teaching profession both theoretically and practically. The theoretical part allows pre-service teachers to learn diverse teaching competences including mathematics teaching methods and the analysis of the mathematics curriculum. The teaching practicum provides an avenue for learning by doing where theories are integrated into practice. It is during the teaching practicum that pre-service teachers demonstrate practical PCK which was investigated in the study. The study contributes to the literature by showing what pre-service teachers might possess and might need in terms of mathematics PCK thereby providing practical implications for teacher education.

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